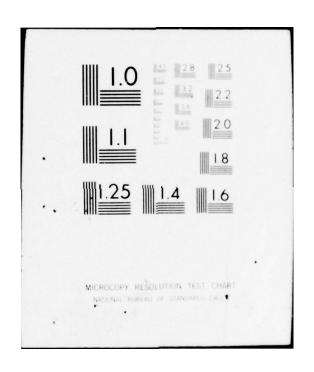
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ON THE UNIQUENESS AND STABILITY OF ENDOCHRONIC THEORIES OF MATERIAL BEHAVIOR Weidlinger Associates

110 East 59th Street New York, New York 10022

October 1976

Topical Report

CONTRACT No. DNA 001-76-C-0127



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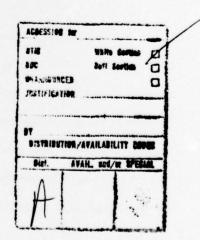
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PREFACE

The author wishes to thank Dr. David Rubin for his useful suggestions regarding the presentation of the material contained in this report.



I INTRODUCTION

Endochronic theories of material behavior were introduced and employed by Valanis, Ref. [4] and [5] to develop a constitutive law for metals which characterizes strain hardening, unloading behavior, cross-hardening (e.g., the effect of pretwist on axial behavior), the alteration of hysteresis loops with continued cyclic straining, and sensitivity to strain rate. Bazant and his co-workers further developed the theory to describe the liquefaction of sand, Ref. [2] and the inelasticity and failure of concrete, Ref. [1].

Fundamentally, the endochronic models do not make use of a yield or loading condition, but instead use a quantity, called the intrinsic time, which is introduced into the constitutive laws of viscoelasticity in place of the real time. The intrinsic time is a monotonically increasing measure of the deformation history of the material. Through its use, substantial similarities in behavior to classical plasticity may be achieved without the introduction of a yield condition. In Ref. [2], [4], [5] the intrinsic time is defined to be independent of real time; this leads to endochronic models which are rate independent.

In this report these endochronic models are examined with respect to Drucker's stability postulate, Ref. [3]. It is demonstrated that the models violate that postulate. This proof is accomplished by a) constructing the simplest possible endochronic model as an example, b) demonstrating that it violates the postulate, and c) showing that the more complex endochronic models exhibit the same qualitative behavior which leads to violation of the postulate for the simple model.

The practical implications of the violation of Drucker's postulate are discussed, leading to the conclusion that the endochronic models are unsuitable for numerical solution of dynamic problems. In this regard, examples are presented of situations involving an endochronic model for which: a) multiple solutions exist for what should be a reasonable physical problem with a unique solution, b) introduction of small errors in initial and/or boundary conditions leads to rapid deterioration in the accuracy of the subsequent computations.

II INSTABILITY OF A SIMPLE ENDOCHRONIC MODEL

The simplest possible endochronic model can be constructed, as in Ref. [1], by starting with a one dimensional (uniaxial stress)

Maxwell model,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{1}{\mathrm{E}} \left[\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\mathrm{Z}} \right] \tag{1}$$

in which t, σ and ϵ are the time, stress and strain, respectively, E is Young's Modulus and Z is the relaxation time of the material. One may rewrite Eq. (1) as

$$d\varepsilon = \frac{1}{E} d\sigma + \frac{\sigma}{ZE} dt$$
 (2)

If the differential of intrinsic time, $d\zeta$, is defined by $d\zeta = |d\varepsilon|$, and the time differential, dt, in Eq. (2) is replaced by $d\zeta$, one obtains the simple endochronic model,

$$d\varepsilon = \frac{1}{E} d\sigma + \frac{\sigma}{ZE} |d\varepsilon|$$
 (3)

This model is rate-independent and exhibits the stress-strain behavior shown in Fig. 1. In particular, continued loading or reloading, of the material results in an asymptotic approach to the limit, or failure, stress, C = ZE, while unloading results in much stiffer behavior than either initial loading or reloading. In fact, the stiffness of this material during reloading is precisely the same as during initial loading at the same stress.

The behavior of the above model will now be examined with respect to Drucker's stability postulate, Ref. [3]. For this purpose, refer to Fig. 2 in

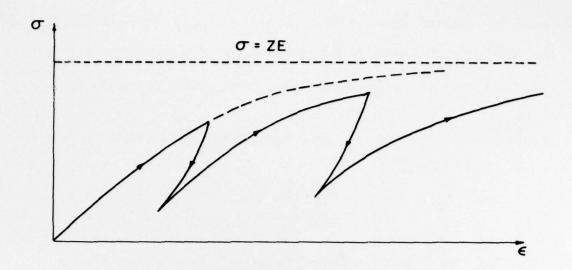


FIG. 1 STRESS STRAIN BEHAVIOR OF A SIMPLE ENDOCHRONIC MODEL

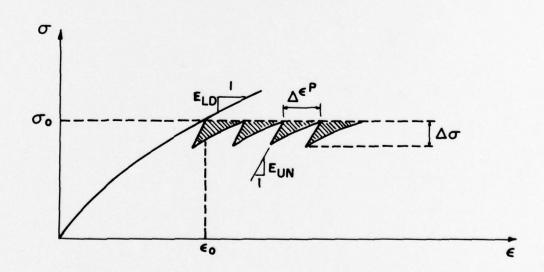


FIG.2 ENERGY EXTRACTION IN INFINITESIMAL UNLOADING - RELOADING CYCLES APPLIED TO ENDOCHRONIC MODELS

which the material, which is initially loaded to an equilibrium state σ_0 , ϵ_0 is subjected to unloading-reloading stress cycles of magnitude $\Delta\sigma$. The inelastic strain produced in each of these cycles is denoted $\Delta\epsilon^p$.

Drucker's stability postulate for a small cycle requires that, for each of the cycles in Fig. 2,

$$\int (\sigma - \sigma_0) d\varepsilon \ge 0 \tag{4}$$

A simple geometric interretation of the integral in inequality (4) shows that its value is equal to the negative of the shaded area in Fig. 2, or $-\Delta\sigma\Delta\epsilon^p/2$, so that inequality (4) is violated. This means that the endochronic model studied here is unstable in the sense that it can be disturbed from an equilibrium state by an external agency which does negative work.

III PRACTICAL IMPLICATIONS

To assess the practical implications of the preceding result, its applicability to more general endochronic models must be established. This can be done quite easily once it is recognized that a fundamental characteristic of all of the previously proposed endochronic models is that, because they do not make use of yield or loading condition, they do not explicitly distinguish between initial loading and reloading, as do work-hardening plasticity theories. The only way to achieve finite differences between loading moduli and reloading moduli in an endochronic model (other than to introduce a functional having the form of a loading or yield surface) is to have an unloading branch of finite intrinsic duration, so that the intrinsic time may change by a finite amount. In any unload-reload cycle of infinitesimal duration and magnitude, however, Druckers stability postulate for a small cycle is necessarily violated.

The demonstrated instability of the endochronic models as proposed in the literature is quite significant because of serious doubts it raises with respect to questions of the uniqueness and continuous behavior of solutions to mechanical problems involving these models (i.e., whether or not such problems are properly posed in the mathematical sense). Related considerations involve the practical problems which inevitably arise whenever one employs a model which can lead to results that are unduly sensitive to the usual, unavoidable small errors that enter into any computation. Especially, in the current era of computer solution of dynamic problems, one must guard against the use of material models that can produce results which are of no value or significance because they are merely consequences of errors introduced by the machine or by the computational scheme from which the results were obtained!

In this regard consider the problem shown in Fig. 3, in which it is required to determine the amount of shortening which occurs when a large weight W is slowly placed on top of a column of length L and area A,whose behavior is described by the endochronic model of Fig. 2. The obvious solution to this problem is that the stress σ_0 in the column is given by

$$\sigma_0 = \frac{W}{A} \tag{5}$$

and the amount of shortening, Δx , is

$$\Delta \mathbf{x} = \mathbf{L} \ \varepsilon_0 \tag{6}$$

in which ε_0 is the strain corresponding to σ_0 on the initial loading curve of Fig. 2. Furthermore, the mass and column are in equilibrium so that Δx remains constant so long as the system is undisturbed.

Now suppose that this problem is being solved on a computer. Inevitably, at some time $t=t_e$, a bit is "dropped", or a round-off error, e, is introduced into the value of Δx . Assuming, for simplicity, that the shortening of the column is small compared to L, the change in the value of the strain is

$$\varepsilon = \frac{\Delta \mathbf{x} - \mathbf{e}}{\mathbf{L}} = \varepsilon_0 - \frac{\mathbf{e}}{\mathbf{L}} \tag{7}$$

and a corresponding decrease in stress is

$$\sigma = \sigma_0 - E_{UN} \frac{e}{L}$$
 (8)

This in turn leads to a downward acceleration, x, of the weight:

$$\frac{W}{g} \ddot{x} = W - A\sigma = AE_{UN} \frac{e}{L}$$
 (9)

As the weight moves downward by an amount x from position (Δx - e) the column stress becomes

$$\sigma = \sigma_0 - E_{UN} \frac{e}{L} + E_{LD} \frac{x}{L}$$
 (10)

so that during the downward motion

$$\frac{W}{g} \ddot{x} + AE_{LD} \frac{x}{L} = AE_{UN} \frac{e}{L}$$
 (11)

where $x = \dot{x} = 0$ at $t = t_e$. Thus the weight moves downward in harmonic motion until the low point, $x_L = 2E_{UN} e/E_{LD}$ is reached. After that point, the weight returns upward, following the equation

$$\frac{W}{g} \ddot{x} + AE_{UN} \frac{x}{L} = AE_{UN} \frac{x_L - e}{L}$$
 (12)

with initial conditions $x = x_L$, $\dot{x} = 0$ at the time the low point was reached. Equations (11) and (12) can be followed up by similar equations describing the subsequent downward and upward motions of the weight. If the series of equations is solved, the resulting column stress-strain behavior is as shown in Fig. 4. In this solution, the stress oscillates about the equilibrium stress σ_0 while the strain drifts towards larger and larger values as time passes. Therefore, two different solutions are obtained depending upon whether or not the very small error e has been made. Further, the average rate at which the column shortens in the "drifting" solution depends on the magnitude of the error e.

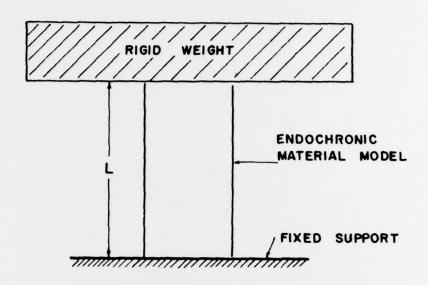


FIG. 3 COLUMN OF ENDOCHRONIC MATERIAL SUPPORTING A RIGID WEIGHT

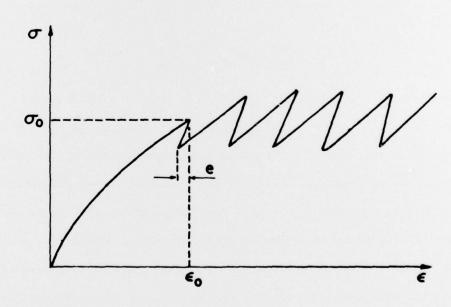


FIG. 4 SOLUTION OF PROBLEM OF FIG. 3 WHEN
A SMALL DISTURBANCE, e, IS INTRODUCED

One might raise the counter argument that because a perfectly plastic material can lead to non-unique displacements under a limit load, the endochronic model is no less stable than a perfectly plastic one. This objection is not valid, however, because the endochronic model is positively unstable, as represented by a negative value of the integral in inequality (4), while the perfectly plastic model possesses neutral stability, as represented by a zero value of the integral in inequality (4). This difference is reflected in the fact that instability of endochronic models occurs not only at limit loads, but over a wide range of stress levels. Furthermore, in displacement controlled problems (which more closely represent the manner in which material models are used in dynamic codes) the endochronic models admit the possibility of stress relaxation due to small high frequency errors, while the perfectly plastic models always give well defined and well behaved stresses in such problems.

As an example of the behavior of the model in displacement controlled situations, consider the problem of Fig. 5, in which a rigid mass rests between two endochronic bars with prestress σ_0 . Any small disturbance, Δx , in the position of the mass leads to the strain disturbance $\Delta \epsilon = \Delta x/L$ which in turn leads to the solution for the stress-strain behavior in one of the bars as shown in Fig. 6. It is apparent from Fig. 6 that a finite amount of stress relaxation results from the infinitesimal disturbance Δx .

The preceeding examples involve single degree of freedom systems only; they do not fully demonstrate the difficulties which can arise if endochronic models are used for more complex problems. In fact, non-uniqueness of solution can occur, as will now be shown by means of an example

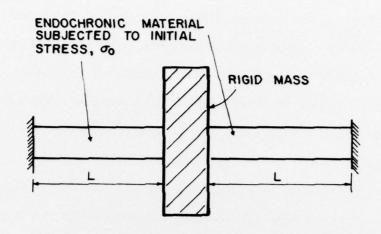


FIG. 5 A PROBLEM WHICH RESULTS IN SPONTANEOUS RELAXATION OF THE STRESS OF FOR AN ARBITRARILY SMALL CHANGE IN THE POSITION OF THE MASS.

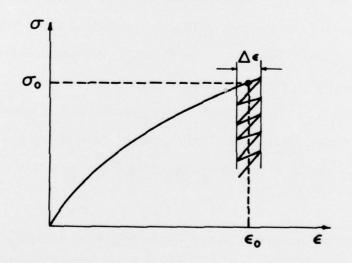


FIG. 6 SOLUTION TO PROBLEM OF FIG. 5 WHEN A SMALL DISTURBANCE, $\Delta \epsilon$, IS INTRODUCED

Consider the problem of an endochronic bar of density of ρ shown in Fig. 7, in equilibrium and at rest under the uniform and constant boundary stress σ_0 (compression positive) at time t = 0. Obviously, the bar may remain at rest under the uniform compressive prestress σ_0 for all t \geq 0. Alternative non-equilibrium solutions will be constructed for this initial and boundary value problem when the bar behaves as indicated in Fig. 2. For this purpose, consider the space-time,or x vs. t, plot of Fig. 8. On this plot, lines of slope \pm $\sqrt{\rho/E_{UN}}$ and \pm $\sqrt{\rho/E_{LD}}$ have been drawn to represent the possible locations of unloading and reloading waves (characteristics) emanating from the generic point 0. These waves travel with the speeds $V_{UN} = \sqrt{E_{UN}/\rho}$ and $V_{LD} = \sqrt{E_{LD}/\rho}$, respectively. Note that Fig. 2 requires $E_{UN} > E_{LD}$ and this in turn implies $V_{UN} > V_{LD}$.

Let candidate solutions to the problem of Fig. 7 be represented by the stresses σ_0 , σ_1 , σ_2 and the velocities \pm v in the regions shown in Fig. 8. (For the trivial solution $\sigma_1 = \sigma_2 = \sigma_0$ and v = 0). For v \geq 0 the equations of motion across the wave fronts give

$$\sigma_1 - \sigma_0 = -\rho V_{UN} v \le 0 \tag{13}$$

$$\sigma_2 - \sigma_1 = \rho V_{ID} V \ge 0 \tag{14}$$

while the strain-displacement relations give

$$\varepsilon_1 - \varepsilon_0 = - v/V_{IIN} \le 0 \tag{15}$$

$$\varepsilon_2 - \varepsilon_1 = \mathbf{v}/\mathbf{V}_{LD} \ge 0 \tag{16}$$

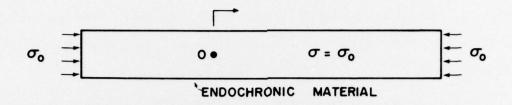


FIG. 7 THE PROBLEM OF AN ENDOCHRONIC BAR INITIALLY AT REST AND AT EQUILIBRIUM UNDER THE PRESTRESS $\sigma_{\rm o}$.

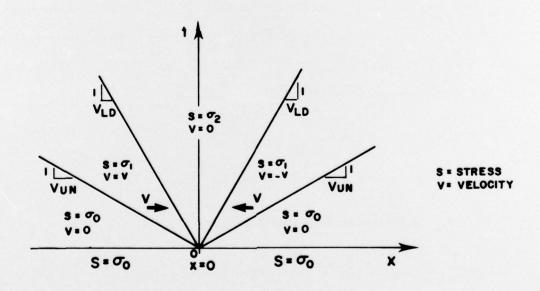


FIG. 8 SOLUTION TO THE PROBLEM OF FIG. 7

Because the region between any pair of wave fronts is uniform in stress and velocity, Eqs. (13 - 16) insure satisfaction of the equations of motion, the strain-displacement relations, the material behavior of Fig. 2, and the apriori assumptions of unloading and reloading at the various fronts in Fig. 8. Therefore, Fig. 8 represents a one parameter family of solutions to the problem of Fig. 7, each member corresponding to a nonnegative value of v. For each nontrivial solution, v > 0, the final stress σ_2 relaxes to

$$\sigma_2 = \sigma_0 - \rho (v_{UN} - v_{LD}) v < \sigma_0$$

over a continually expanding region containing point 0. Simultaneously, the resulting loss in strain energy is continually converted into the kinetic energy, $\rho v^2/2$, of the material in the expanding region between the unloading and reloading wave fronts. Because of the multiple solutions represented by Eqs. (13 - 16), it is clear that endochronic models will not always lead to unique solutions to physically meaningful continuum problems.

It should be noted that it is not necessary to introduce an error, e, into the continuum problem in order to produce a new solution. However, it is clear that if some error e did exist at the space time point 0, it would propagate over an expanding region of material without significantly dissipating (as the velocity "error" v does in Fig. 8). Assuming that one had some basis for deciding which of the many solutions to a problem was the correct one, the way in which the errors would propagate in a numerical solution can be used to obtain an estimate of the length of time one may run an endochronic problem on a computer before the solution is destroyed by such errors.

All numerical schemes for solving dynamic continuum problems begin

with a set of interpolating or approximating functions which describe the conditions at time t for the computation to $t + \Delta t$. In general, these functions cannot exactly represent conditions at any time t, so that errors are introduced at most space points at most time steps. Let us denote by e the average relative error introduced at any space-time point in the value of any unknown q. If we assume e to be a normally distributed random variable, then the average error arising from a single space point over N time steps is e \sqrt{N} . Because each error leads to an expanding region of errors of spatial extent on the order of $(v_{UN}^{}$ - $v_{LD}^{})^{N \wedge t}$, some number of points on the order of $N(1 - V_{ID}/V_{IIN})$ will contribute to the average error in the value of q at any space point after N time steps. Because the error due to each of these points is also random, a relative error on the order of eN $\sqrt{1 - V_{LD}/V_{UN}}$ can be expected in q. For most practical interpolation schemes e is rarely less than $10^{-3}\,$ so that N < 500 is required for errors less than 50% in one dimensional situations whenever there is a significant difference between the unloading and reloading wave speeds. In two dimensional situations the error region expands in two directions so that approximately N² points contribute to the error at any point, and the total relative error is on the order of ${ t N}^{3/2}$ e. This means fewer than 100 time steps can be carried out before the solution completely degenerates. Because many practical problems require much larger numbers of time steps for adequate solution, and because such computations are easily achieved on modern computers, it is clear that the error propagation characteristics of endochronic models make such models unsuitable for the numerical solution of mechanical problems.

IV CONCLUSION

A stability analysis of current endochronic models, which represent the differences between loading and unloading behavior without resorting to the concept of a yield condition, has been performed using Drucker's stability postulate in the small for a cycle. It has been shown that these endochronic models violate that postulate, and the practical implications of such a violation have been discussed. Analysis of some simple problems has illustrated the difficulties which can arise when endochronic models are used, and leads to the conclusions that such models are unsuitable for the numerical solution of mechanical problems.

Although it is possible to circumvent the difficulties which arise in the current endochronic models, it appears that this would require the introduction into the models of a function or functional having the same features as the yield condition or loading surface of classical plasticity. Whether the resulting model could be called endochronic is not a technical issue but a matter of semantics.

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